A new principle of construction of logical machines

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In a logical machine, i. e. a machine which is constructed to deal with problems about propositions or their truth values, a device is needed for representing the truth values " true " and " false ". The most common method of representation of truth values is based on *voltage states of a single pole* ; e. g., the truth values " true " and " false " are represented by means of a live pole and a grounded pole, respectively. In this case, in order to compute, e. g., the truth value of a conjunction of six propositions out of their truth values, a black box is needed, containing six input poles as well as an output pole and such that its output pole is live if and only if all input poles are live, and in the opposite case, its output pole is grounded. Such a black box, or a " connective box for conjunction ", can be easily designed using a relay and six rectifiers shunted by means of resistors ; see e. g. [1]. Similarly, connective boxes for the other operations of the propositional calculus can be designed (rectifiers being needed, besides conjunction, in the case of disjunction and implication only, moreover, resistors in the case of conjunction only) ; see again [1].

On the other hand, it is well known that the representation of the truth values by means of the *conductivity states between two poles*, viz., that of the values " true " and " false " by means of a closed and an open current-path, respectively, shows some advantages over the representation by means of voltage states. Indeed, using the former method of representation (i. e. that by means of the conductivity states between two poles), the truth value of the conjunction and disjunction of propositions can be computed by means of wiring alone, viz. by means of serial and parallel connection, respectively. However, the same cannot be done for non-isotonous operations of the propositional calculus, such as e. g. negation, implication and equivalence. Hence, a logical machine based on the representation of truth values by means of the conductivity states between two poles requires for the computation of the truth values of propositions formed by means of these operations some devices containing relays or similar elements as well.

Now, in a lecture delivered at the International Congress of Mathematicians in Edinburgh last month, I have proved that, using an appropriate representation by means of *conductivity states between more than two poles*, the result of any operation in a finite algebra, in the sense of Universal Algebra, can be computed by wiring alone. I do not intend to formulate here this theorem in its exact mathematical form, nor to give its proof in general ; however, I shall sketch the proof in the case of a two argument operation A o B in the set of the two truth values. In this case, the representation used in the general proof reduces to a representation of the truth values " true " and " false " by means of two conductivity states, appropriately chosen from the five possible conductivity states between three poles, as shown on Fig. 1 ; they correspond to

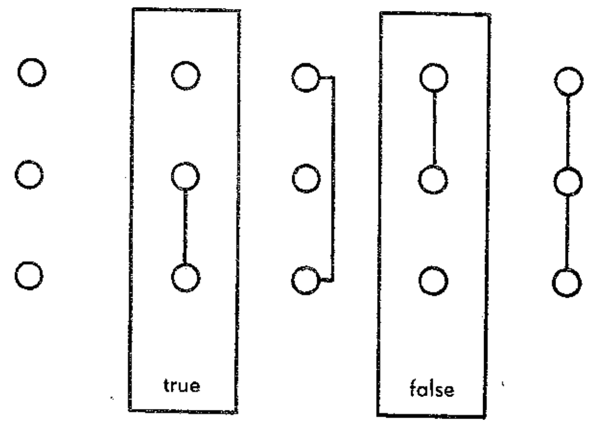

FIG. 1.

sockets, the holes of whose are connected with the springs of a change over contact. Now, let us take a wire box with three input plugs of three terminals each, as well as with an output socket of three holes and connect them first as shown on Fig. 2, i. e., connect the middle output hole S_0 with the middle input terminal A_0, further, connect the lower and upper input terminals A_1 and A_2 with the middle input terminals B_0 and B'_0, respectively. Moreover, connect the remaining input terminals B_1, B_2, B'_1 and B'_2 with the output holes corresponding to the truth values true o true, true o false, false o true and false o false respectively, where S_1 is said to correspond to the truth value "true" and S_2 to the truth value "false".

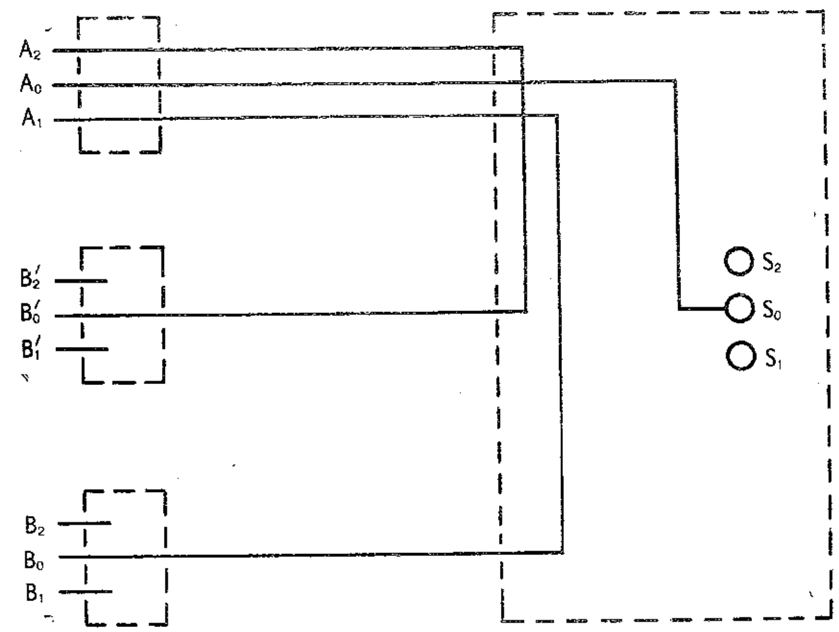

FIG. 2.

Then, for arbitrary truth values A and B, inserting the input plugs into three hole sockets corresponding to the truth values A, B and B again, but otherwise insulated each from the others, we get the conductivity state of the output socket, corresponding to the truth value A o B. Of course, the two lower three terminal plugs can be united into one six terminal plug. Thus, e. g. in the case of the operation A←→B (equivalence), we get the wire-box shown on Fig. 3.

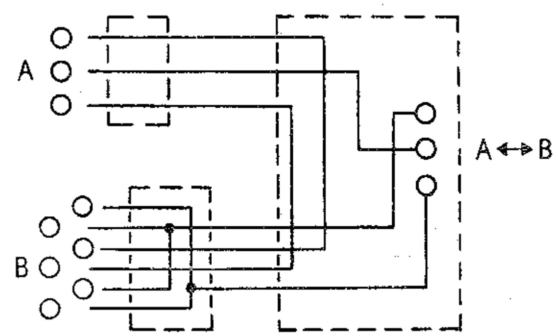

FIG. 3.

Provided, for one of the values of A, the value A o B does not depend on B (which is the case for the operations $A \wedge B$ (conjunction), $A \vee B$ (disjunction), as well as $A \rightarrow B$ (implication)), we have to short-circuit the middle terminal of one of the lower plugs with one of its lower on upper terminals; hence, we can omit this plug at all. Thus, e. g. in the case of the operation $A \vee B$, we get the wire-box shown on Fig. 4.

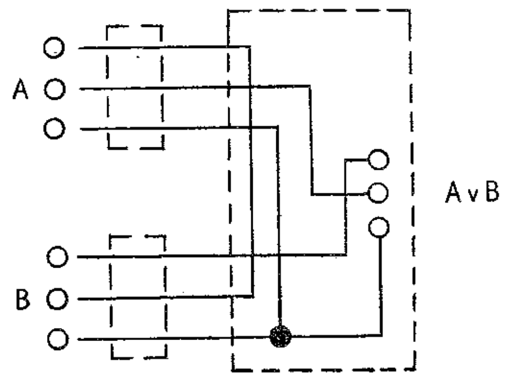

FIG. 4.

By the way, using the identities $A \rightarrow B = \bar{A} \vee B$, $A \wedge B = \overline{\bar{A} \vee \bar{B}}$ and $A \oplus B = \bar{A} \leftrightarrow B$ (where $\bar{A}$ denotes negation and $A \oplus B$ denotes symmetrical difference), further the fact, that in case of using symmetrical plugs and sockets, negation can be performed by means of reversed plugging, the disjunction box of Fig. 4 can be used for

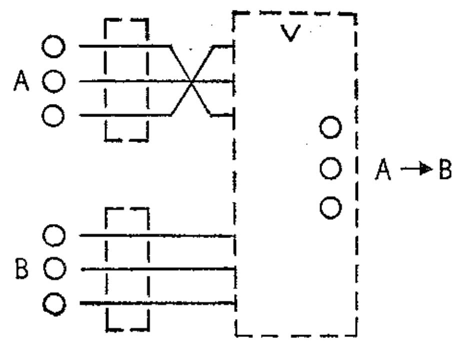

FIG. 5.

implication and conjunction as well, as shown on Fig. 5 and 6,

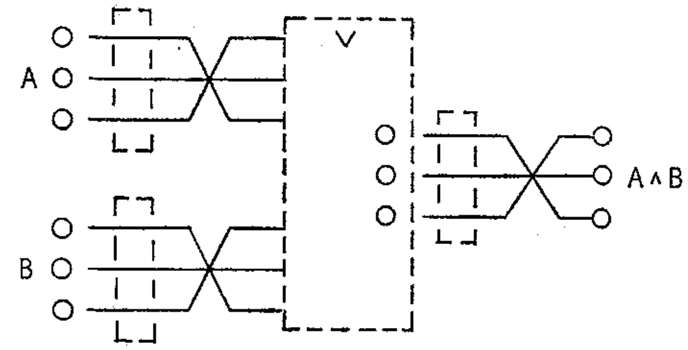

FIG. 6.

respectively, further, the equivalence box of Fig. 3 can be used for symmetrical subtraction as well, as shown on Fig. 7.

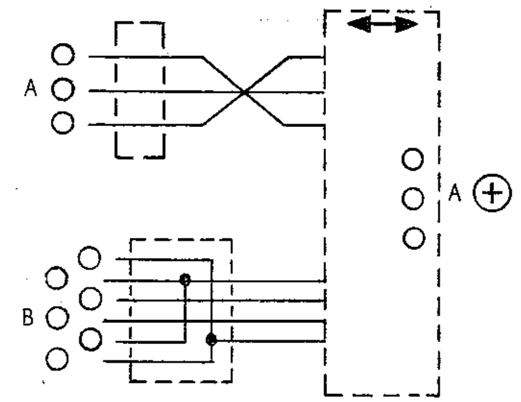

FIG. 7.

By means of our connective boxes, containing nothing but wiring, a logical machine has been constructed in Szeged for computing truth tables of arbitrary formulae of the propositional calculus (containing, in the actual form of the machine, at most 8 logical variables). For this purpose, " variator relays " controlling several change over contacts, one of whose is used for indicating purposes whereas the springs of the others are connected with the holes of sockets corresponding to logical variables, are induced, by means of a control unit formed of relays and rotary switches, to scan over all possible combinations of their released and operated states. For each formula to be investigated, a corresponding circuit is composed by means of inserting the input plugs of the connective boxes, corresponding to the logical operations figuring in the formula, into sockets corresponding to the arguments of these operations. The truth value of the formula itself is indicated by means of the last output socket as shown on Fig. 8. Of course, an electronical version of this logical machine is also possible.

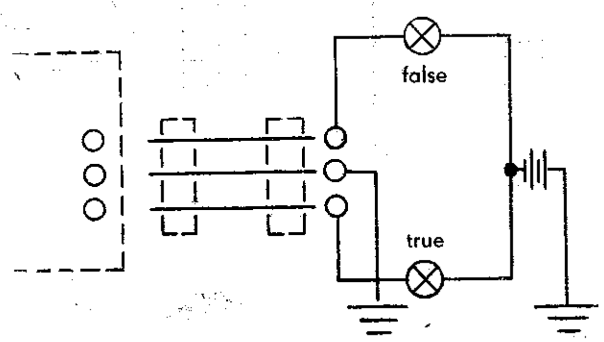

Fig. 8.

Similarly, using the fact that arithmetical operations can be reduced to logical ones, viz. to those of the propositional calculus, a digital computer can be constructed whose arithmetical unit does not contain anything but wire.

REFERENCES.

[1] McCallum, D. M. and Smith, J. B. *Mechanical reasoning*, Electronic Engineering, April 1951, pp. 126-133.